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10MAT41

Fourth Semester B.E. Degree Examination, July/August 2021
Engineering Mathematics - IV

Time: 3 hrs. Max. Marks:100

Note: *Answer any FIVE full questions.*

1 a. Find y at x = 1.02, given dy = (xy − 1) dx and y = 2 at x = 1 applying Taylor's series method (considering terms upto fourth degree). **(07 Marks)**

b. Use Picard's method to obtain the second approximate solution. Given $\frac{dy}{dx} = x - y^2$ with y(0) = 1 and compute y(0.1) correct to four decimal places. **(07 Marks)**

c. Solve $\frac{dy}{dx} = 3x + \frac{y}{2}$ with y(0) = 1 and step size 0.1 using modified Euler's method at x = 0.1. (Perform three iterations). **(06 Marks)**

2 a. Use Runge – Kutta method of fourth order to solve $\frac{dx}{dt} = y - t$, $\frac{dy}{dt} = x + t$ given x(0) = 1, y(0) = 1. Compute x(0.1) and y(0.1). **(07 Marks)**

b. Apply Picard's method to obtain the third approximation to the solution of $\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} - 6y = 0$. Given that when x = 0, y = 1, $\frac{dy}{dx} = 0.1$. **(07 Marks)**

c. Employ Milne's method to compute y(0.8). Given that $\frac{d^2y}{dx^2} = 1 - 2y\frac{dy}{dx}$ and y(0) = 0, y(0.2) = 0.02, y(0.4) = 0.0795, y(0.6) = 0.1762, y'(0) = 0, y'(0.2) = 0.1996, y'(0.4) = 0.3937, y'(0.6) = 0.5689. **(06 Marks)**

3 a. Define Analytic function and derive Cauchy Riemann equations in polar form. **(07 Marks)**

b. Construct the analytic function whose real part is $e^{2x}(x\cos 2y - y\sin 2y)$. **(07 Marks)**

c. If u and v are harmonic functions, show that $\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right) + i\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)$ is analytic. **(06 Marks)**

4 a. Find the Bilinear transformation which maps z = ∞, i, 0 into w = -1, -i, 1 and also find the invariant points. **(07 Marks)**

b. Discuss the transformation $w = z^2$. **(07 Marks)**

c. Evaluate $\int_c \frac{e^{2z}}{(z+1)(z-2)} dz$, where c is the circle |z| = 3. **(06 Marks)**

5 a. Find the value of $J_{1/2}(x)$ and $J_{-1/2}(x)$. **(07 Marks)**

b. Obtain the solution of Legendre's differential equation
$$(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + n(n+1)y = 0.$$ **(07 Marks)**

c. Express $x^3 + 2x^2 - 4x + 5$ in terms of Legendre polynomials. **(06 Marks)**

6 a. State and prove Baye's theorem of probability. **(07 Marks)**

b. If A and B are events with $P(A) = \frac{3}{4}$, $P(B) = \frac{1}{5}$, $P(A \cap B) = \frac{1}{20}$, find $P(A \cup B)$, $(A \cap \overline{B})$, $P(\overline{A} \cap \overline{B})$, $P(A \mid B)$. **(07 Marks)**

c. An office has 4 secretaries handling respectively 20%, 60%, 15%, 5% of the files of all government reports. The probability that they misfile such reports are respectively 0.05, 0.1, 0.1 and 0.05. Find the probability that the misfiled report can be blamed on the first secretary. **(06 Marks)**

7 a. The probability distribution of a finite random variable X is given by the following table :

x	-3	-2	-1	0	1	2	3
P(x)	k	2k	3k	4k	3k	2k	k

Determine the value of k, $P(x \le 1)$, $P(x > 1)$, $P(-1 < x \le 2)$. **(07 Marks)**

b. Obtain the mean and variance of Poisson distribution. **(06 Marks)**

c. The marks of 1000 students in an examination follows a normal distribution with mean 70 and standard deviation 5. Find the number of students whose marks will be i) less than 65 ii) more than 75 iii) between 65 and 75. Given that $P(0 < z < 1) = 0.3413$. **(07 Marks)**

8 a. Explain the terms i) Null hypothesis ii) Type I and Type II errors iii) Test of significance. **(07 Marks)**

b. A group of 10 boys fed on a diet A and another group of 8 boys fed on a different diet B for a period of 6 months recorded the following increase in weights (ℓbs).

Diet A :	5	6	8	1	12	4	3	9	6	10
Diet B :	2	3	6	8	10	1	2	8		

Test whether diets A and B differ significantly regarding their effect on increase in weight. ($t_{.05} = 2.12$ for 16 d.f) **(07 Marks)**

c. A die is thrown 264 times and the number appearing on the face follows the following frequency distribution.

x	1	2	3	4	5	6
f	40	32	28	58	54	60

Calculate the value of χ^2. **(06 Marks)**

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